AMS 206

1

this time: Bayes's Theorem for propositions

next time: for real-valued unknowns

read: (T) ch. 1, 2, App. A, B; (G) ch. 1

unknown

A = (this patient is HIV+)

$\theta = \begin{cases} 1 & \text{if } A \text{ is true} \\ 0 & F \end{cases}$

$D = \begin{cases} 1 & \text{if ELISA says } + \\ 0 & - \end{cases}$

$\mathbb{P}$ = this guy's HIV status

$\mathcal{Q}$ = is A true?

$\mathcal{C}$ =

$\mathcal{B} = (B_1, \ldots, B_k)$

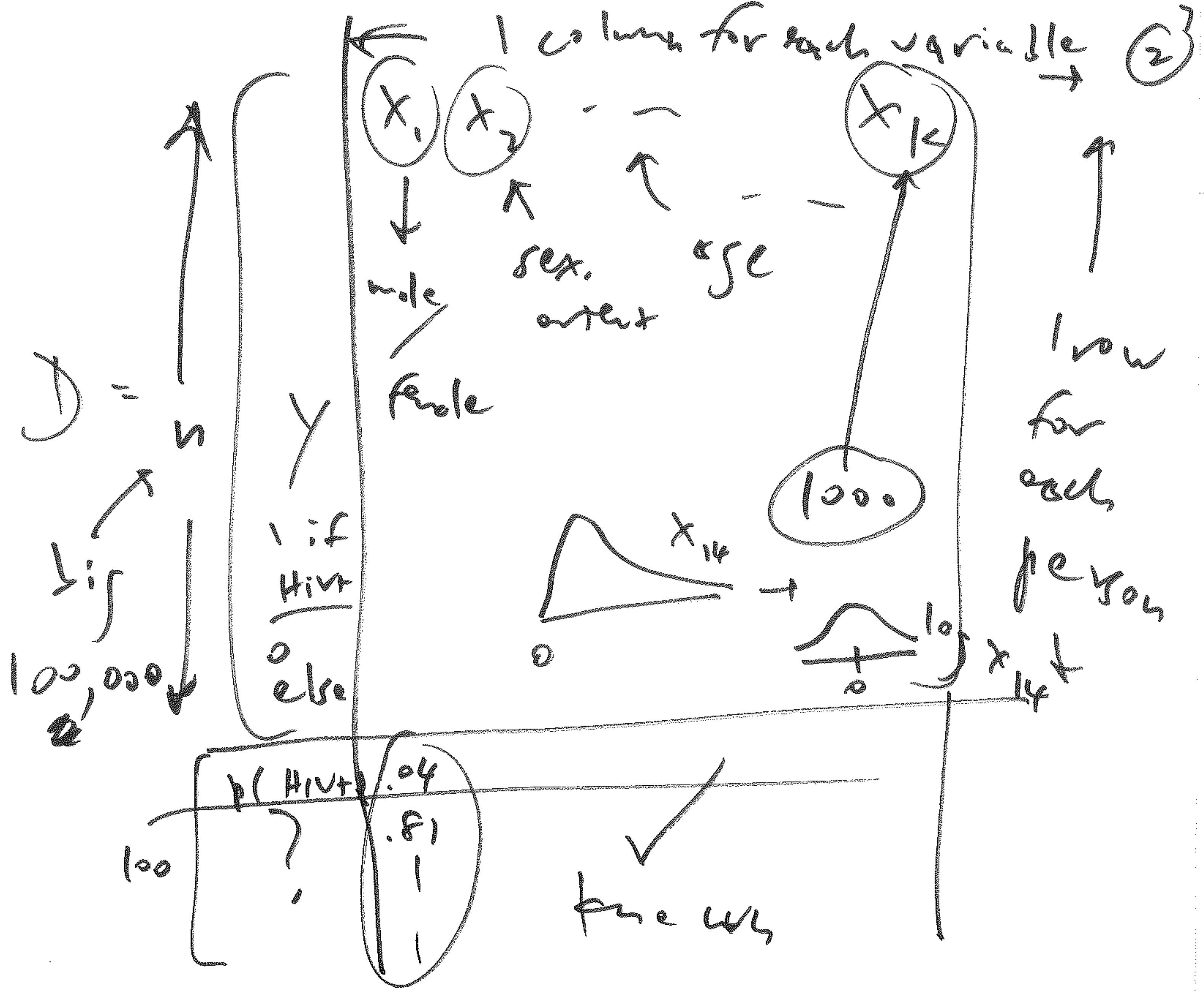
P^*

all people similar to this patient in all relevant ways

symptoms

like at random

[this guy]



(A)ssumptions

Information (context $C \Leftrightarrow B$)

(J)udgments $p(\theta = 1 \mid D = 1)$ and B

$\rightarrow p(\theta = 1 \mid D = 1,$ assumptions & judgments$)$

$P(A) = 0.01$

prevalence of HIV in people like this guy

want

$P\left(\begin{matrix} \text{really} \\ \text{is HIV+} \end{matrix} \middle| \begin{matrix} \text{ELISA} \\ \text{says} \\ \text{HIV+} \end{matrix} \right)$

this to be big (close to +1)

(+) = ELISA says HIV +

(−) = ——— −

A = really is HIV +

not A = ——— HIV−

we want to compute $= P(A \mid +) = ?$

want close to 1

$P(+ \mid A) = 0.95$ ← sensitivity

$P(- \mid \text{not } A) = 0.98$ ← specificity

want close to 1

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③ ways to compute $P(A \mid +)$ from page 3 numerical facts:

(I) 2×2 contingency table

ELISA really & says the truth

really is HIV+ / HIV+

what ELISA says

	A	not A	
+	95	198	293
−	5	9,702	
	100	9,900	10,000

① $P(A) = .01$

② $P(+ \mid A) = .95$

③ $P(- \mid \text{not } A) = .98$

9702

← constrained by prevalence

$$P(A \mid +) = \frac{95}{293} = \boxed{0.32}\,④232\cancel{08}\ \cancel{1911263}$$

driven by specificity

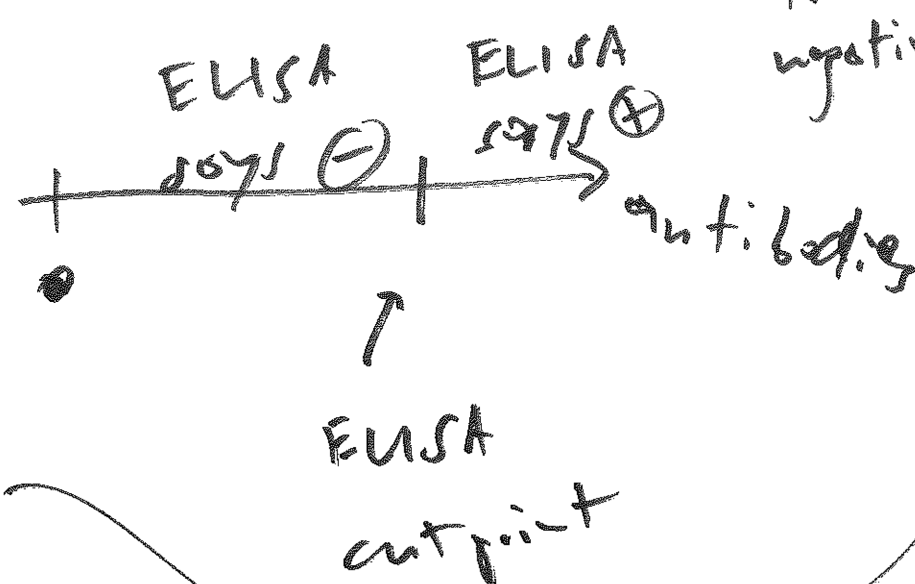
$P(\text{really is HIV+} \mid \text{ELISA says HIV+}) = 0.32$ (!) (terrible) truth

true positive

		A	not A
ELISA says	+	right	false positive error
	−	false negative error	right

worse (a) (b)

true negative



- the patient
- the doctor
- blood bank

$$P(\text{false} +) = P(\text{not}A \mid +) = \frac{198}{293} \doteq 68\%$$

$$P(\text{false} -) = P(A \mid -) = \frac{5}{9707} \doteq 0.05\%$$

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III

$$P(A \mid +) = \frac{\checkmark P(A) \cdot \checkmark P(+ \mid A)}{P(+)}$$

annoying normalizing constant

not yet known to us

Bayes's Thm. directly

$P(A) = .01$

$P(+ \mid A) = .95$

$P(- \mid \text{not } A) = .98$

II Bayes's Theorem in odds form

$$O = \frac{P(A \mid +)}{P(\text{not } A \mid +)} = \frac{\left[\dfrac{P(A) \cdot P(+ \mid A)}{\cancel{P(+)}}\right]}{\left[\dfrac{P(\text{not } A) \cdot P(+ \mid \text{not } A)}{\cancel{P(+)}}\right]}$$

p

odds ratio $= \dfrac{p}{1-p}$

$$= \left(\frac{P(A)}{P(\text{not } A)}\right) \left(\frac{P(+ \mid A)}{P(+ \mid \text{not } A)}\right)$$

$$\left(\frac{P(A \mid +)}{P(\text{not } A \mid +)}\right) = \left(\frac{P(A)}{P(\text{not } A)}\right) \cdot \left(\frac{P(+ \mid A)}{P(+ \mid \text{not } A)}\right)$$

(posterior odds ratio in favor of A given +) = (prior odds ratio in favor of A being true) · (Bayes factor or likelihood ratio in favor of A given +)

$$= \left(\frac{.01}{.99}\right) \cdot \left(\frac{.95}{1-.98}\right)$$

= (99 to 1 odds ratio against true HIV+) · (47.5 to 1 odds ratio in favor of true HIV)