This time: Metropolis & Gibbs sampling; Big-Data Data Science

Metropolis sampling

AMS 206
20 Mar 18
1

$k = 1$

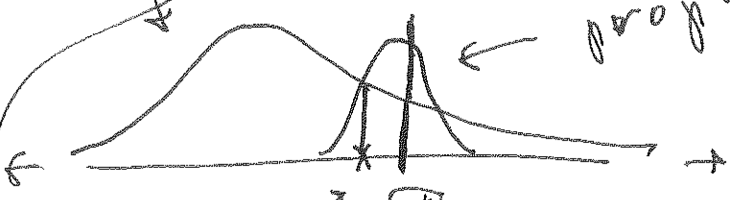
must be symmetric

PDSD σ

target $p(\theta \mid y)$

$q(\theta)$ proposal distribution

$N(\theta_t^*, \underline{\underline{\sigma}}^2)$



$\boxed{\theta_t^*}$

$\theta^*_{proposed}$

this is an <u>uphill</u> move

rule 1: accept all uphill moves

$$\boxed{\frac{p(\theta^*_{proposed} \mid y)}{p(\theta_t^* \mid y)}} = r(\theta^*_{proposed}, \theta_t^*)$$

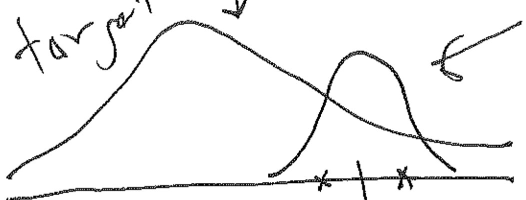
$\underline{\underline{\theta^*_{t+1} = \theta^*_{proposed}}}$

$p(\theta \mid y) =$

$c\, p(\theta) \cdot \ell(\theta \mid y)$

target

proposal distribution



θ_t^* $\theta^*_{proposed}$

$\cancel{p(\theta^*_{proposed} \mid y)}$

$$\frac{\cancel{c}\; p(\theta^*_{proposed}) \cdot \ell(\theta^*_{proposed} \mid y)}{\cancel{c}\; p(\theta_t^*) \cdot \ell(\theta_t^* \mid y)}$$

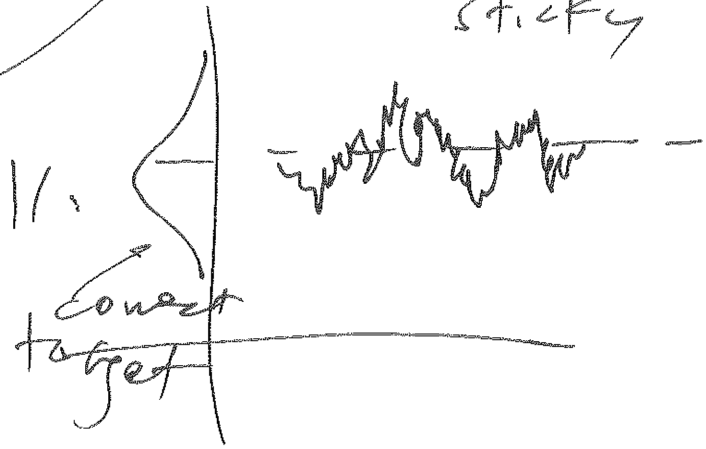
rule 2: accept a downhill move with probability $r(\theta^*_{proposed}, \theta_t^*)$

$\theta^*_{t+1} = \theta^*_{proposed}$

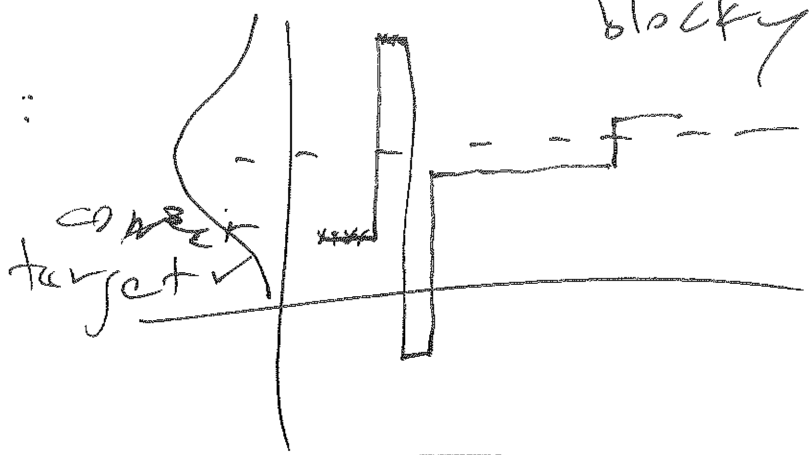
rule 3: if reject under rule 2,

$$\theta^*_{t+1} = \theta^*_t$$

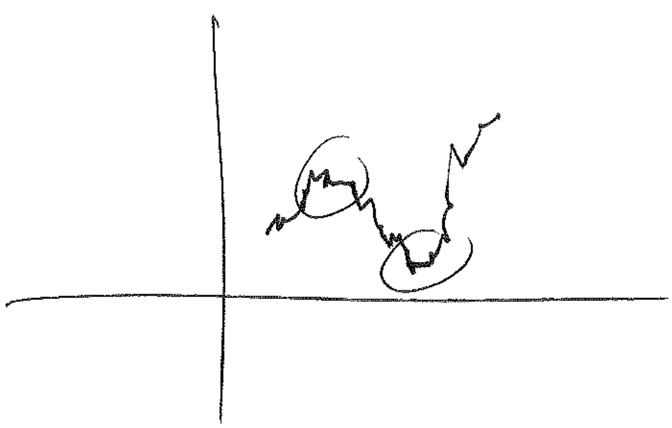
if PDSD σ is too small:



if PDSD σ is too big:

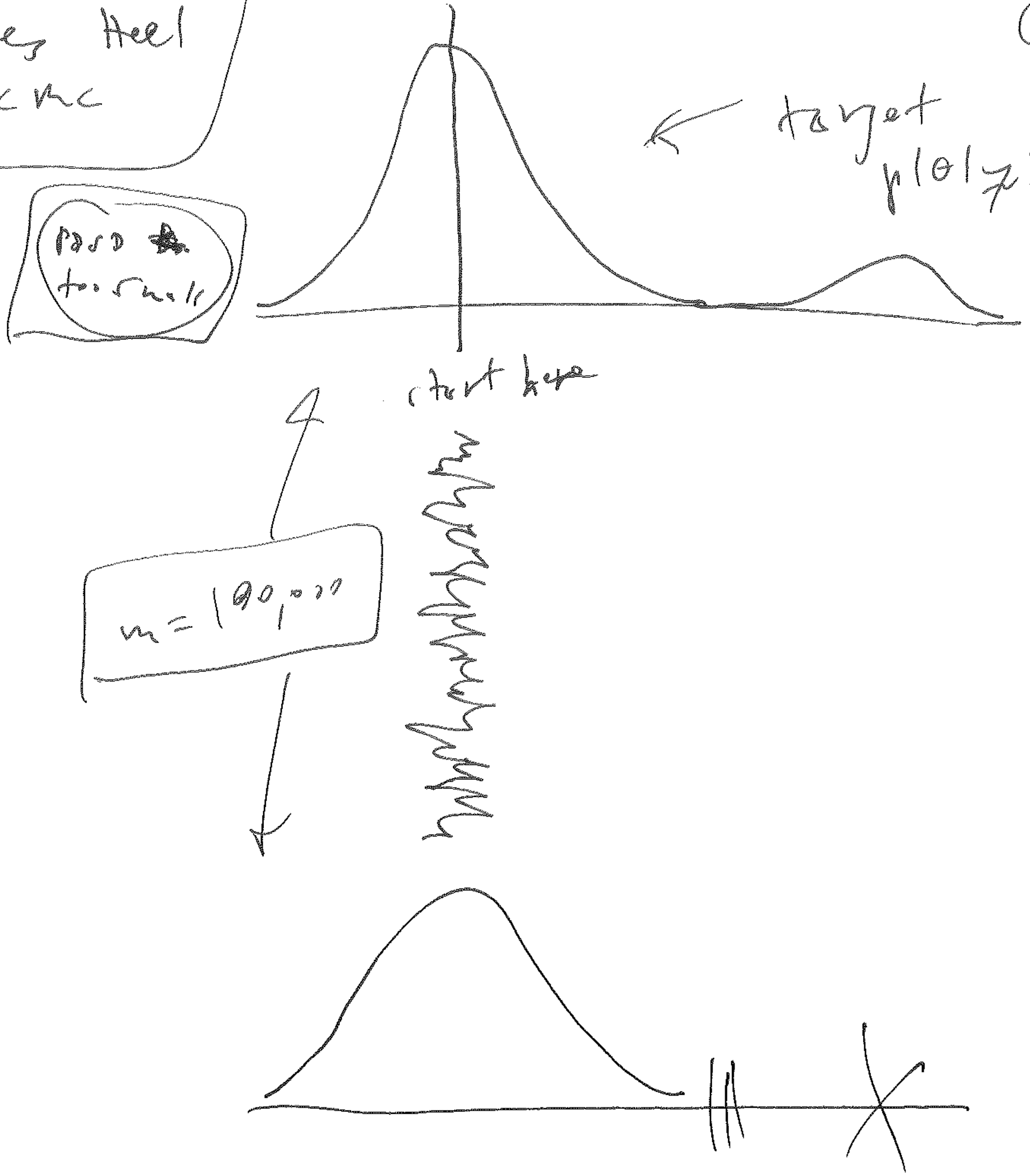


with optimal PDSD



keep track of ~~rejection~~ acceptance rate of algorithm $\hat{\pi}$; optimal PDSD $\Rightarrow$ $\hat{\pi} \doteq 0.45$ $(k=1)$

Achilles Heel of MCMC



Gibbs sampling

NB10 t model $\underset{\sim}{\theta} = (\mu, \sigma, \nu)$

we want $p(\mu, \sigma, \nu | y\ B..)$; hard

Suppose can sample from

$$\left\{\begin{array}{l} p(\mu \mid y\ \sigma\ \nu\ B \ldots) \\ p(\sigma \mid y\ \mu\ \nu\ B \ldots) \\ p(\nu \mid y\ \mu\ \sigma\ B \ldots) \end{array}\right\}$$

full-conditional dist. of $p(\underset{\sim}{\theta} \mid y\ B \ldots)$

intuitively

(1.1) sample μ_1^*

from $p(\mu \mid y\ \sigma_0\ \nu_0)$

(1.2) sample σ_1^*

from $p(\sigma \mid y\ \mu_1^*\ \nu_0)$

(1.3) sample ν_1^*

from $p(\nu \mid y\ \mu_1^*\ \sigma_1^*)$

	μ	σ	ν
0	μ_0	σ_0	ν_0
1	μ_1^*	σ_1^*	ν_1^*
		σ_2^*	

(2.1)
(2.2) repeat
(2.3)

⋮ repeat

acceptance rate 1