this time: decision theory

next time: unknown $\theta \in (0,1)$

read: J (ch 1, 2; Appendices A, B) new: J $\boxed{\text{ch } 3}$; G (ch. 1)

new: $\boxed{\text{G ch. 2}}$

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ELISA case study (continued)

$$\left(\begin{array}{l}\text{posterior} \\ \text{odds} \\ \text{ratio} \\ \text{in favor} \\ \text{of } A \text{ given } \oplus\end{array}\right) = o = \left(\frac{1}{99}\right)\left(\frac{47.5}{1}\right) = \frac{95}{198}$$

$$\left(o = \frac{p}{1-p} \right) \leftrightarrow p = \frac{o}{1+o} = \frac{\frac{95}{198}}{1+\frac{95}{198}} = \frac{95}{293} \doteq 0.32$$

III compute $P(A \mid +)$ directly with Bayes's Thm.

$$P(A|+) = \frac{\check{P}(A) \cdot \check{P}(+|A)}{P(+) \leftarrow ?}$$

DV Lindley "extending the conversation"

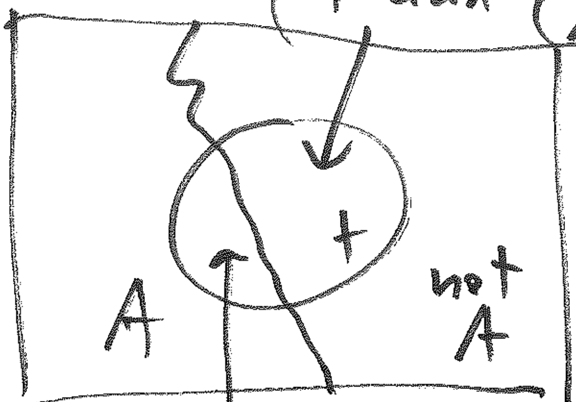
data

$$P(+) = P(+ \text{ and } A) + P(+ \text{ and } (\text{not } A))$$

$\{A, \text{not } A\}$

by partitioning on another dimension

(+ and not A)



(+ and A)

$$P(+ \text{ and } A) = P(A) \cdot P(+|A)$$

$$P(+ \text{ and } (\text{not } A)) = P(\text{not } A)\, P(+|\text{not } A)$$

Law of total probability

$$P(A|+) = \frac{P(A) \cdot P(+|A)}{P(A) \cdot P(+|A) + P(\text{not } A) \cdot P(+|\text{not } A)}$$

$$* \quad \frac{(.01)(.95)}{(.01)(.95) + (.99)(\underbrace{1-.98}_{.02})}$$

$P(A) = .01$

$P(+ \mid A) = .95$

$P(- \mid \text{not } A) = .98$

$$= \frac{95}{95 + 198}$$

$$= \frac{95}{293} \doteq 0.32$$

decision theory

possible actions ← action space $(a \mid \theta)$

a_1 = (run ELISA: if ELISA says $\oplus$, declare (really is $\oplus$); if ELISA says $\ominus$, declare (really is $\ominus$))

a_3 = (run WB; if WB says $\oplus$, declare A; if WB says $\ominus$, declare (not A))

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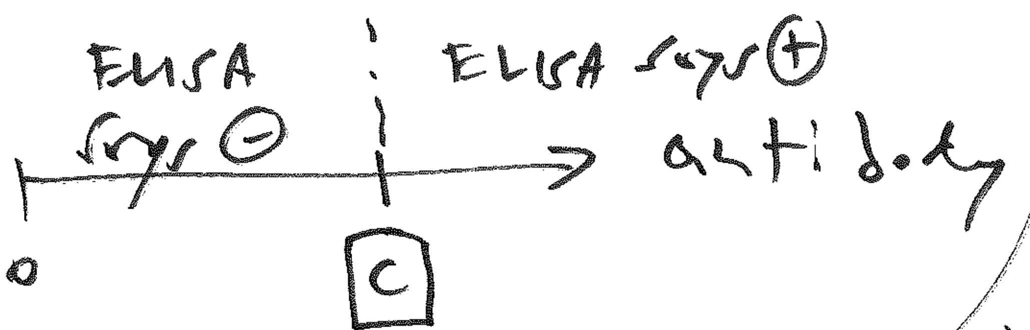
a_2 = (run ELISA: if $\oplus^{\text{ELISA}}$, run WB: if WB $\oplus \to$ A
if $\ominus$ (not A);
if ELISA $\ominus$, declare (not A))



have pretend
$(a \mid \mathcal{B}) = \{a_1, a_2\}$

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we now have to specify a utility function

$$U(a, \theta \mid \mathcal{B}) \in \mathbb{R}^1$$

action unknown

WLOG assume big U better than small U ⑤

$U(a, \theta \mid \mathcal{B}) = f($costs, benefits) of action a if θ is the actual unknown

if θ were known, optimal a would maximize $U(a, \theta \mid \mathcal{B})$

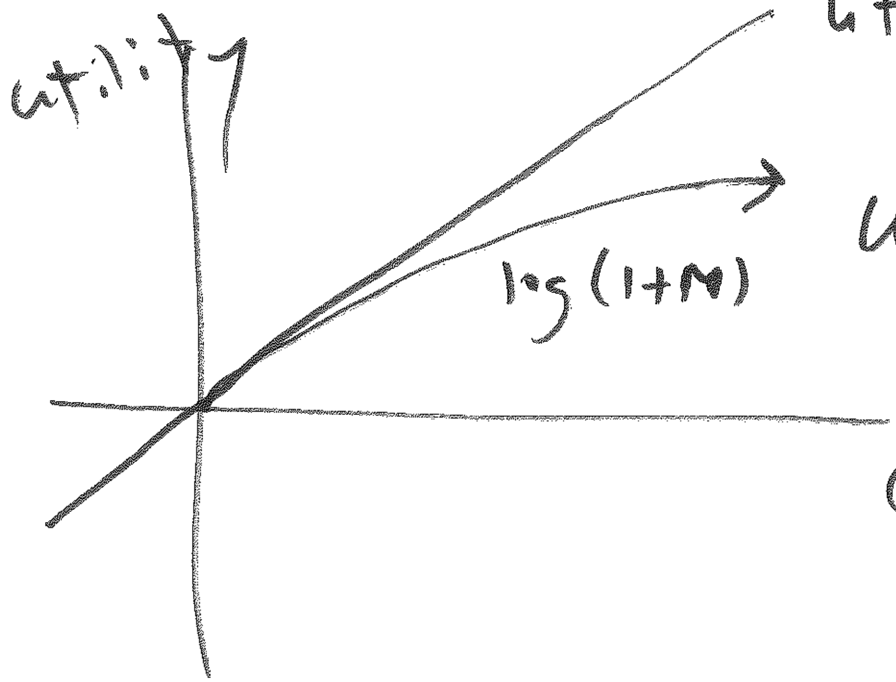
$$a^* = \underset{a \in (\mathcal{A} \mid \mathcal{B})}{\text{argmax}} \; U(a, \theta \mid \mathcal{B})$$

optimal a

But can't do this because we don't know θ

utility:

Daniel Bernoulli (1720)



utility = money

$U(a, \theta \mid B) \approx$ money

money M

↓

$U(M) = \log(1+M)$

$U(a, \theta \mid B) = f($costs, benefits$)$

from action a

if unknown $= \theta$

$= \boxed{g(\text{costs})} + h(\text{benefits})$

monetary

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prefer a_2 to a_1 if

$$+.0197\, L_{II} - .0205\, L_I$$

$$-.0293\, c_2 > 0$$

pop $\mathcal{P}$ $\boxed{?}$

stat. inf.

sample $\mathcal{S}$

the observed AMI patients

1 = Y
0 = N

die?

y_1 0
0
1
0
1
⋮
y_n 0

$n = 400$

die?

$N = ?$ big

1s & 0s

$\boxed{\text{like}}$ at random (SRS) = IID

mean $\theta = ?$

fixed unknown scalar

$$S_n = \sum_{i=1}^{n} y_i = 72$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{S_n}{n} = \frac{72}{400} = 0.18 = 18\%$$

$$P_F(|\bar{y} - \theta| \le .01) = ?$$

high

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inference: $\bar{y}$ $(= 18\%)$ is a good guess (estimate of) for θ; but how good?

$$\mathcal{D} \text{ (data)} = \underset{\sim}{y} = (y_1, \dots, y_n)$$