AMS 206
27 Feb 18

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this time: MLE for vector $\underset{\sim}{\theta}$

next time: simulation-based computation (MCMC

read: (as before)

take-home test 2 (THT2) target 6 Mar, drop dead 17 Mar

THT3 will be published around 10 Mar, drop dead due 23 Mar

probably 1 more quiz

$k = 1$ $\quad \frac{\partial}{\partial \theta_1} \ell\ell(\theta_1 \mid \underset{\sim}{y}\, \mathcal{B}) = 0$, solve to get $\hat{\theta}_{1\,MLE} = \hat{\theta}_1$

$$\hat{I}(\hat{\theta}_1) = \left[-\frac{\partial^2}{\partial \theta_1^2} \ell\ell(\theta_1 \mid \underset{\sim}{y}\, \mathcal{B}) \right]_{\theta_1 = \hat{\theta}_1}$$

$$\hat{V}_{RS}(\hat{\theta}_1) = \frac{1}{\hat{I}(\hat{\theta}_1)} = \hat{I}^{-1}(\hat{\theta}_1)$$

$$\hat{SE}_{RS}(\hat{\theta}_1) = \sqrt{\hat{I}^{-1}(\hat{\theta}_1)}$$

approx 95% CI for θ_1:

$$\hat{\theta}_1 \pm 1.96\, \hat{SE}(\hat{\theta}_1)$$

Gaussian (CLT)

$k > 1$

$$\left[\begin{array}{c} \frac{\partial}{\partial \theta_1} \ell\ell = 0 \\ \vdots \\ \frac{\partial}{\partial \theta_k} \ell\ell = 0 \end{array} \right]$$

solve to get $\underset{\sim}{\hat{\theta}}_{MLE}$ (analytically or numerically)

$$\underset{k \times k}{\hat{\Sigma}(\hat{\theta})} \text{ covariance matrix} = \underset{k \times k}{\hat{I}^{-1}} \quad \leftarrow \text{matrix inverse}$$

$$\underset{k \times k}{\hat{I}(\hat{\underset{\sim}{\theta}})} = - \begin{array}{c|c} & \theta_1 \quad \cdots \quad \theta_k \\ \begin{array}{c} \theta_1 \\ \vdots \\ \theta_k \end{array} & \begin{bmatrix} \frac{\partial^2}{\partial \theta_1^2} & \frac{\partial^2}{\partial \theta_1 \theta_2} & \cdots \\ \frac{\partial^2}{\partial \theta_2 \theta_1} & \ddots & \\ & \cdots & \frac{\partial^2}{\partial \theta_k^2} \end{bmatrix} \end{array} (\theta = \hat{\theta})$$

symmetric $(k \times k)$ matrix

$ll(\underset{\sim}{\theta} \mid y)$

Hessian of $ll(\underset{\sim}{\theta} \mid y)$

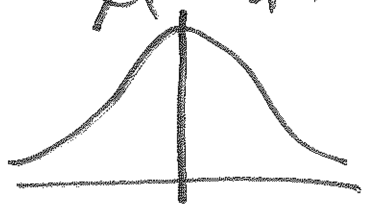
$$\hat{\Sigma} = \begin{array}{c|c} & \hat{\theta}_1 \quad \cdots \quad \hat{\theta}_k \\ \begin{array}{c} \hat{\theta}_1 \\ \vdots \\ \hat{\theta}_k \end{array} & \begin{bmatrix} \hat{V}(\hat{\theta}_1) & \hat{C}(\hat{\theta}_1, \hat{\theta}_2) & \\ & V(\hat{\theta}_2) & \vdots \\ & \ddots & \\ & & \hat{V}(\hat{\theta}_k) \end{bmatrix} \end{array}$$

$$\hat{SE}(\hat{\theta}_j) = \sqrt{\hat{V}(\hat{\theta}_j)}$$

j^{th} diagonal element

approx 95% CI : $\hat{\theta}_j \pm 1.96 \, \hat{SE}(\hat{\theta}_j)$

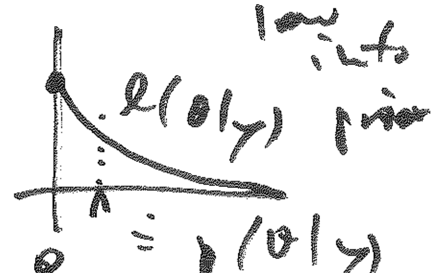
$\ell(\theta|y)$



$\doteq p(\theta|y)$

large n, low info prior

small n, low info prior



$\doteq p(\theta|y)$

$(\theta > 0)$

max (for MLE) = mean (for Bayes)

Fisher $\doteq$ approx Bayes

$\hat{\theta}_{MLE} = 0$

& yet all of $p(\theta|y)$ is to the right of 0

Fisher (max) $\neq$ Bayes (integrating)

$p(\underset{\sim}{\theta}_{1 \times k} \,|\, y\,\mathcal{B} \dots)$

k large

want to approximate this

ex.: NB10

$(\mu\,\sigma\,\nu\,\mathcal{B}) \sim$ diffuse

$(Y_i \,|\, \mu\,\sigma\,\nu\,\mathcal{B}) \stackrel{IID}{\sim} t_\nu(\mu, \sigma^2)$

$(i = 1, \dots, n)$

$\underset{\sim}{\theta} = (\mu, \sigma, \nu)$

Monte Carlo dataset

$p(\underset{\sim}{\theta} \,|\, y\,\mathcal{B} \dots)$

time	μ	σ	ν	σ^2
1	μ_1^*	σ_1^*	ν_1^*	$(\sigma_1^*)^2$
2	μ_2^*	σ_2^*	ν_2^*	.
.	.	.	.	.
.	.	.	.	.
m	μ_m^*	.	.	.
	$\bar{\mu}^*$			

(100,000)

want big

want IID draws

(white noise)

draw $(\mu_1^*, \sigma_1^*, \nu_1^*)$ at random from $p(\underset{\sim}{\theta} \,|\, y\,\mathcal{B} \dots)$

m = # of monitoring draws

$$E_{RS}(\bar{\mu}^*) = p(\mu \mid y B \cdots)$$, i.e., $\bar{\mu}^*$ is unbiased for

$$\bar{\mu}^* = \frac{1}{m} \sum_{j=1}^{m} \mu_j^*$$

$\mu_j^* \sim IID$

$$V_{RS}(\bar{\mu}^*) = \frac{\sigma_\mu^2}{m} \neq V(\mu \mid y B \cdots)$$

$$\widehat{MCSE}(\bar{\mu}^*) = \sqrt{\hat{V}_{RS}(\bar{\mu}^*)}$$

$$= \frac{\hat{\sigma}_\mu}{\sqrt{m}}$$

sample SD of μ_j^* draws