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AMS 206: Quiz 2 *[21 points, plus 4 extra credit points]*

Name: ____

Please supply your answers to the questions below in the spaces provided. If your answers extend to more than three pages, please ensure that each continuation answer identifies the question it's answering on the extra page(s), and (if you're using the scanning option for submission) make sure to scan all pages of your solution for uploading to `canvas.ucsc.edu`.

(business) A big problem facing banks and merchants alike is determining potential customers' credit-worthiness. You've probably had the experience of trying to use a credit card to pay for something — store owners typically have a little electronic machine they pass your card through, and information stored in the magnetic strip on the back (mainly your credit card account number) is transmitted over a telephone line to a central *credit-verification system* somewhere. This system uses a computer program to decide if your purchase should be approved, based on factors like how often it thinks your card has been used lately and how recently it thinks you've paid your bill. If you pass this invisible screening, a little green light goes on back in the store, and you walk out with your purchase.

Of course, any computer-based system of this kind makes mistakes sometime, because of faulty information or bad programming: sometimes credit cards that are good are declared bad, sometimes vice versa. Events of this type look random to the people at the bank trying to figure out why they happen — at least until the causes of the mistakes are determined — so it makes sense to talk about the *probability* that a bad credit card is declared good, and the probability that a good card is judged bad. Standard terminology is to call the kind of mistake in which, given that the system says a card is bad, it's really good a *false positive*, and the other kind of mistake — in which, given that the system says a card is good, it's actually bad — a *false negative*. (Evidently the people who made up this terminology were thinking of "positive" in this context as equivalent to "calling a credit card bad" and "negative" as amounting to "calling the card good," which is a little perverse, but there it is.)

People evaluate the quality of credit-screening systems of this type by running tests in which (a) they attempt a number of fake "purchases" with some credit cards that are known to be good and some others that are known to be bad, and (b) they look to see how often the system gets it right. Suppose that in one test of this type on the system we're going to look at, 97% of the test "purchases" with cards known to be good were labeled good by the system, and 99% of the "purchases" in which the test cards were bad were declared bad by the system. Suppose further that the system is to be used in a market in which about 0.5% of all attempted purchases are with bad credit cards.

Somebody now walks into a store using this credit verification system and tries to make a purchase by credit card, and the system comes back with a negative opinion about this person's credit-worthiness.

(1) Let G stand for the proposition that (the card really is good); let $+$ signify that (the system says the card is bad); and let $-$ stand for the proposition (the system says the

card is good). Express the three numerical facts (0.5%, 97%, 99%), mentioned two paragraphs above, in unconditional and conditional probability terms using the symbols $\{G, \text{not } G, +, -\}$. *[3 points]*

(2) Using any one of the methods examined in class for making the following computation — the (2×2) contingency table (if you want nice integers, imagine 100,000 total transactions); Bayes's Theorem in odds form; and/or computing the denominator using the Law of Total Probability) — and briefly explaining each step of your solution, show that the conditional probability the card is indeed bad, given the system says it's bad, is only about 14%! *[6 points] Extra credit [4 additional points]:* Make the computation using the other two methods, thereby verifying that they all give the same answer.

(3) Express the propositions (false positive) and (false negative) in conditional probability terms using the symbols $\{G, \text{not } G, +, -\}$, and show that the false positive and false negative rates of the system under the circumstances described are 86% and 0.005%, respectively. *[4 points]*

(4) Now let's think decision-theoretically. There are three actors in this drama: the customer, the store owner/merchant and the bank that issued the credit card. Describe the real-world consequences of both false-positive and false-negative errors for each of these actors (i.e., your answer should describe 6 consequences in it, 2 for each of the 3 actors). *[6 points]*

(5) From the viewpoint of the bank, and using Your answers in parts (3) and (4), does the 14% result in part (2) mean that the people who designed the screening system on behalf of the bank are stupid? If not, why not? Explain briefly. *[2 points]*